GENERALIZED KINETIC MODEL OF THE CREEP AND RUPTURE STRENGTH OF A STRAIN-HARDENING MATERIAL

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The author of [1] proposed a generalized kinetic model of the creep and rupture strength of metals in the absence of strain-hardening:

$$\dot{p} = Bf(\sigma)\exp(qa),\ p(0) = 0,$$
$$\dot{a} = L\varphi(\sigma)\exp(\varkappa a),\quad a(0) = a_0,\quad a(t_*) = a_*. \tag{1}$$

Here, p is the creep strain; σ, stress; a, structure parameter; t_*, time to failure; B, L, q, $\varkappa$, a_0, and a_*, material constants; $q \geq 0$, $B \geq 0$, $L \geq 0$; the functions f and φ are determined to within the constant multiplier; the dots denote differentiation with respect to time t; the temperature is assumed to be fixed.

Model (1) is a generalization of four classical models of the second and third stages of creep [1]. It does not describe the strain-hardening stage, which usually conforms satisfactorily to strain-hardening theory:

$$\dot{p} = f_1(\sigma)p^{-\lambda},\ p(0) = 0 \tag{2}$$

(λ is a material constant).

There are two adequately substantiated theories which consider the effect of strain-hardening. The strain relation [2, 3]

$$\dot{p} = f_1(\sigma)p^{-\lambda}\exp(\alpha p),\ p(0) = 0 \tag{3}$$

(α is a material constant). The energy relation [4-6]

$$\dot{A} = f_2(\sigma)A^{-\lambda}/\Psi(A), \tag{4}$$

where $A = \int_0^t \sigma dp$ is the specific energy dissipated during creep; at $\lambda = 0$ (absence of strain-hardening effect), $\Psi(A) = (A_* - A)^{\alpha}$; A_* and α are material constants.

In the present study we propose to generalize (1). At the first stage of creep, this generalization would be equivalent to strain-hardening theory (2) and would contain Eqs. (3) and (4) as special cases.

The validity of (4) was checked experimentally during standard loading in the presence of strain-hardening. In this instance, during the stage of transient creep [$\Psi(A) = \text{const}$], model (4) is equivalent to the strain-hardening theory. Under nonsteady loading conditions, (2) conforms to the experimental results more closely than does the energy model in [7, 8]. The effects sometimes seen with a stepped or momentary increase in load are complex in nature and cannot be described either by strain-hardening theory or by models of the type (4) [8]. Thus, to describe the nonsteady section in model (1), it is necessary to introduce a second structure parameter – creep strain. Then, with allowance for the similarity of the creep curves and Eq. (2), model (1) takes the form

$$\dot{p} = Bfp^{-\lambda_1}\exp(qa),\quad \dot{a} = L\varphi p^{-\lambda_2}\exp(\varkappa a)$$

(λ_1 and λ_2 are material constants). In order for model (4) to be a special case of this relation, it is necessary to set $\lambda_1 = \lambda_2 = \lambda$. We finally obtain

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$$\dot{p} = Bf(\sigma)p^{-\lambda}\exp(qa),\ p(0) = 0,$$
$$\dot{a} = L\varphi(\sigma)\,p^{-\lambda}\exp(\varkappa a),\quad a(0) = a_0,\quad a(t_*) = a_*. \tag{5}$$

For the first stage ($a = a_0$), model (5) is equivalent to (2).

Let us examine special cases of model (5).

1. $q = \kappa (r = \infty)$. Then, $a = a_0 + zL/B\left(z = \int_0^t \frac{\varphi(\sigma)}{f(\sigma)}dp\right)$ and model (5) can be written as

$$\dot{z} = D\varphi(\sigma)p^{-\lambda}\exp(\alpha z),\ z(0) = 0\ (\alpha = qL/B,\quad D = B\exp(qa_0)). \tag{6}$$

With $f \equiv \varphi (z \equiv p)$, models (6) and (3) are equivalent. Under the conditions of a constant stress, $p = (f/\varphi)z$, and Eq. (6) has the form

$$\dot{z} = D\Phi(\sigma)z^{-\lambda}\exp(\alpha z),\ z(0) = 0\quad (\Phi(\sigma) = [f(\sigma)]^{-\lambda}[\varphi(\sigma)]^{1+\lambda}). \tag{6'}$$

When $q \neq \kappa$, in Eqs. (5) it is convenient to make the substitution of variables $l = \operatorname{sgn}(r)\frac{B}{L(q-\varkappa)}\exp\{a(q-\varkappa)\}$:

$$\dot{p} = Df(\sigma)\,p^{-\lambda}l^r,\quad \dot{l} = \operatorname{sgn}(r)\,D\varphi(\sigma)\,p^{-\lambda}l^r,\quad l(0) = l_0,\quad l(t_*) = l_*, \tag{5'}$$

where $r = \frac{q}{q-\varkappa}$; $D = B\left[\operatorname{sgn}(r)\frac{Lq}{Br}\right]^r$; $\operatorname{sgn}(r) = \begin{cases} 1, & r > 0, \\ -1, & r < 0. \end{cases}$

2. $q < \kappa (r \leq 0)$, for this $l = l_0 - z$ and (5) is written in the form

$$\dot{z} = D\varphi(\sigma)p^{-\lambda}(l_0 - z)^{-|r|}. \tag{7}$$

For $\sigma = \text{const}$, Eq. (7) is changed to

$$\dot{z} = D\Phi(\sigma)z^{-\lambda}(l_0 - z)^{-|r|}. \tag{7'}$$

With $\varphi(\sigma) = \sigma f(\sigma)(z \equiv A)$, model (7') corresponds to energy model (4).

3. $q > \kappa (r \geq 0)$. Then $l = l_0 + z$, and model (5) takes the form

$$\dot{z} = D\varphi(\sigma)p^{-\lambda}(l_0 + z)^r. \tag{8}$$

With a constant stress,

$$\dot{z} = D\Phi(\sigma)z^{-\lambda}(l_0 + z)^r. \tag{8'}$$

In the general case, in model (5) it is necessary to find five material constants (two of the constants – such as L and κ – can be chosen arbitrarily [1]): D, λ, r, l_0, and z_*, where $z_* = z(t_*)$. It is also necessary to find the two functions $f(\sigma)$ and $\varphi(\sigma)$ in this model. The constant D and the function $\varphi(\sigma)$ are determined from the rupture-strength curve $t_* = t_*(\sigma)$. The function $f(\sigma)$ is found from the condition of similarity of the curves $[\varphi(\sigma)/f(\sigma)]p$ with different stresses [in particular, it is found from the condition $(\varphi/f)\cdot p(t_*) = \text{const}$]. The constants λ, r, and l_0 are obtained from the corrected creep curve $t/t_* = V(p, \lambda, r, l_0)$.

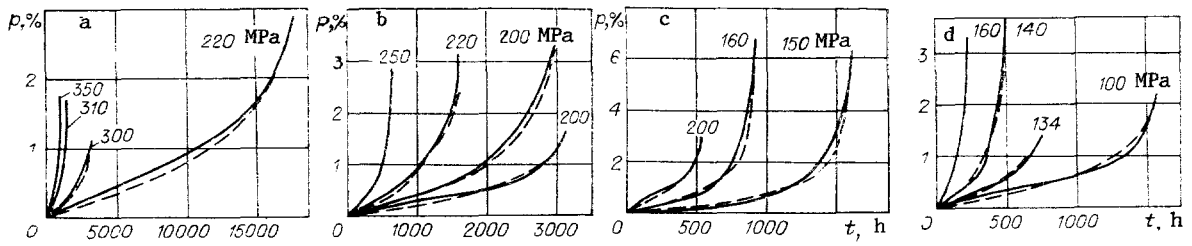

Fig. 1

TABLE 1

T, °C	λ
750	0,6
800	0,3
850	0,12
880	0,06

TABLE 2

T, °C	σ, MPa	H, %	c	T, °C	σ, MPa	H, %	c
750	220	1,028	0,9405	850	150	0,809	0,9997
	300	0,4017	0,9353		160	1,035	0,9986
	310	0,4025	0,9854		200	1,183	0,9086
	350	0,52	0,9641				
800	200	0,5263	0,9573	880	100	0,7155	0,9528
	200	1,017	0,9622		134	0,67	0,9748
	220	0,869	0,9727		140	0,8384	0,9871
	250	0,747	0,9764		160	0,872	0,977

To simplify the calculations, it is important to have an analytic expression for the function V. This is possible only for integral values $\lambda = 1, 2, \ldots$. As the calculations we performed showed, this statement is valid for $\lambda > 1$. At $\lambda < 1$, the section of transient creep is not usually pronounced and can be ignored if allowance is made for the third section. As an example, the solid lines in Fig. 1 show data from [9] on the creep of heat-resistant nickel alloy ÉI826. Table 1 shows values of the constant λ found from the initial sections of creep curves. As a first approximation, the experimental data are satisfactorily described by Eq. (6) with $\lambda = 0$. Here, the creep strain [1]

$$p = H(\sigma)\ln(1 - ct/t_*)^{-1}$$
$$(H(\sigma) = (1/\alpha) f(\sigma)/\varphi(\sigma), \quad c = 1 - \exp(-\alpha z_*)). \tag{9}$$

Theoretical values of H and c are shown in Table 2, where the temperature values correspond to Fig. 1a-d. The dashed lines, constructed from Eq. (9), satisfactorily describe the experimental results.

In accordance with model (5), in the general case we write the creep curves in the form

$$t/t_* = [J(z) - J(0)]/[J(z_*) - J(0)], \tag{5''}$$

where $J(z) = \begin{cases} \int z^\lambda \exp(-\alpha z)\, dz, \\ \int z^\lambda (l_0 - z)^{|r|}\, dz, \\ \int z^\lambda (l_0 + z)^{-r}\, dz \end{cases}$

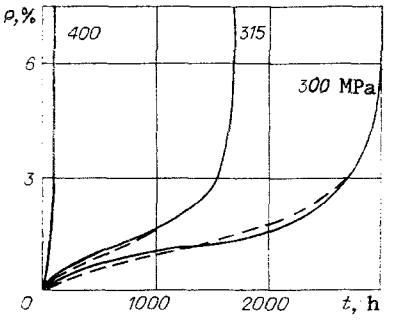

Fig. 2

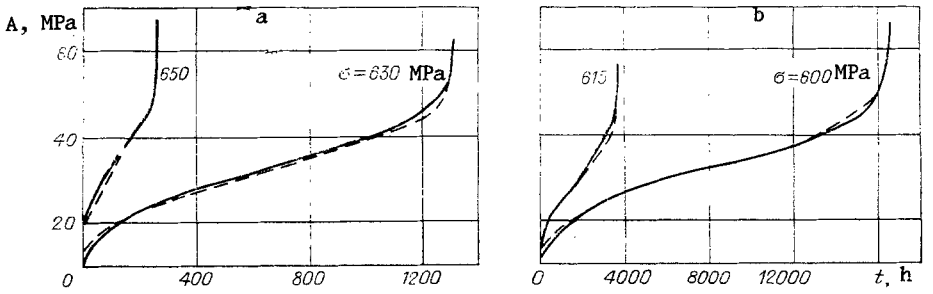

Fig. 3

for cases 1-3, respectively. Integrating these relations with $\lambda = 1, 2, \ldots$, we obtain the following for cases 1-3:

$$J(z) = -\exp(-\alpha z)\sum_{k=0}^{\lambda} J_k(z)/\alpha^{k+1}; \tag{6''}$$

$$J(z) = (l_0 - z)^{|r|+1}\sum_{k=0}^{\lambda}\frac{J_k(z)\,l_0^k}{(\lambda+|r|+1)(\lambda+|r|)\ldots(\lambda+|r|-k+1)}; \tag{7''}$$

$$J(z) = \begin{cases} (l_0+z)^{1-r}\sum\limits_{k=0}^{\lambda}(-1)^k\dfrac{J_k(z)\,l_0^k}{(\lambda-r+1)(\lambda-r)\ldots(\lambda-r-k+1)}, & r\neq 1, \\ \sum\limits_{k=0}^{\lambda-1}(-1)^k\dfrac{J_k(z)\,l_0^k}{\lambda(\lambda-1)\ldots(\lambda-k)} + (-1)^{\lambda}\,l_0^{\lambda}\ln(l_0+z), & r=1. \end{cases} \tag{8''}$$

Here, $J_k(z) = \begin{cases} z^{\lambda}, & k=0, \\ \lambda(\lambda-1)\ldots(\lambda-k+1)\,z^{\lambda-k}, & k=1, 2, \ldots, \lambda. \end{cases}$

In the general case, it is necessary to find three constants in Eq. (5''): λ, r (or α), and ℓ_0; λ is determined from the sections of transient creep in accordance with (2), while the other two are determined numerically. In particular, when the number of constants can be reduced to two [1] – cases 1 and 2 with $\ell_0 = z_*$, case 3 with $\ell_0 = 0$, etc. – both constants can be determined numerically in accordance with Eqs. (5'')-(8'').

Figure 2 (T = 565°C) shows the creep curves of steel ÉP44 [3]. The creep of ÉP44 is satisfactorily described by the strain model (z ≡ p). The value of r turns out to be fairly large ($|r| \approx 10$), so as the theoretical value we took $r = \infty$. With $\lambda = 1$, we write Eqs. (5'') and (6'') in the form

$$\frac{t}{t_*} = \frac{1/\alpha^2 - \exp(-\alpha p)\,(p/\alpha + 1/\alpha^2)}{1/\alpha^2 - \exp(-\alpha p_*)\,(p_*/\alpha + 1/\alpha^2)}. \tag{6'''}$$

It can be seen from the dashed lines in Fig. 2, constructed from (6''') with $p_* = 7.5\%$ and $\alpha = 1.3$ that the agreement with the experimental results (solid lines) is satisfactory.

Figure 3 shows creep curves of titanium alloy 3V at room temperature [4] (20°C). Here, the energy model z ≡ A is realized. It is evident from the calculations that the creep process is described by case 2 with $\lambda = 5$. In this instance, it can be approximately assumed that $\ell_0 = A_*$ [1], and Eqs. (5'') and (7'') are changed to:

$$\frac{t}{t^*} = 1 - (1 - A/A_*)^{|r|+1}\sum_{k=0}^{5}\frac{(|r|+1)\ldots(|r|+k)}{k!}\left(\frac{A}{A_*}\right)^k. \tag{7'''}$$

The dashed lines constructed from (7''') with $r = -4.8$ coincide satisfactorily with the solid lines (experiment).

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DESIGN AND LAYOUT OF LAYERED PLATES

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Calculation of the stiffness characteristics of layered plates is carried out on the basis of asymptotic studies for the problem of elasticity theory in thin (thickness $h \to 0$) regions [1]. Application to the equations obtained of methods in [2, 3] made it possible to solve the problem of designing layered plates with a prescribed set of stiffness characteristics.

Characteristic Equations for Layered Plates. An asymptotic analysis was provided in [1] for the problem of elasticity theory in a thin region whose thickness h tends toward zero, and two methods were proved which may be used in designing layered plates: a limiting transition with $h \to 0$ and the same limiting transition invoking a cellular problem (problem L in the terms of [1]). In the first case we obtain explicit equations for calculating stiffness and, in the second, the same equations but with prior solution of the cellular problem. In this work we follow the second path in studying the mechanics of layered plate bending.

Let the plate in question be formed of layers of uniform isotropic materials (parallel planes Ox_1x_2). Plate thickness $h \ll 1$. We cover the plate with a rectangular network with a side ~h long. An element of this network P_h separates a cell $Y_h = P_h \times [-h/2, h/2]$, called the cellular periodicity. In variables $y = 2x/h$ a cell of periodicity Y_h is converted into

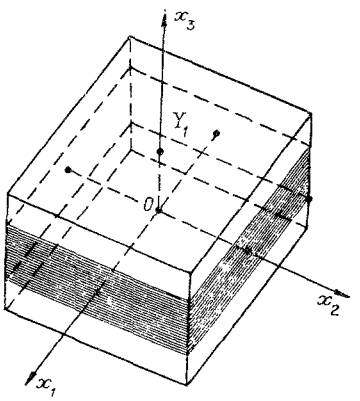

Fig. 1

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